Workshop on "Recent Developments in Algebra"

(July 08, 2024 - July 10, 2024)

Titles and Abstracts

DEPARTMENT OF APPLIED SCIENCES
INDIAN INSTITUTE OF INFORMATION TECHNOLOGY ALLAHABAD
PRAYAGRAJ (211015) INDIA

Some problems on multiplicative Lie algebra and commutator identities
Prof. Ramji Lal, IIIT Allahabad

Abstract: TBA

Word maps on groups
Prof. Anupam Kumar Singh, IISER Pune

Abstract: Given a word w in d variables it gives rise to a map on a group G, called word maps defined by evaluation. The main question is to understand what is the fibre and image of such maps. Such questions are motivated by the Waring problem in number theory and Ore's conjecture in group theory. In the last couple of decades, there has been tremendous progress in this area. In this expository talk, we will see some of the results proved in this direction and the kind of questions being studied.

Closure Properties of the Non-Abelian Tensor Product and its Applications
Prof. Guram Donadze, Georgian Tech University

Abstract: I am going to talk on the closure properties of the non-abelian tensor product of groups. I and my collaborators have observed that if a class of group X is such that
(i) non-abelian tensor product is closed on X;
(ii) X is closed under taking an epimorphic images,
then X is a Schur class. Using the technique of the non-abelian tensor product one can prove the analogue of Schur-Baer theorem for finitely generated groups. I will also talk on the applications of the non-abelian tensor product in non-abelian homological algebra.

Seshadri constants
Prof. Krishna Hanumanthu, CMI Chennai

Abstract: Seshadri constants of line bundles on projective varieties were defined by J.-P. Demailly who was inspired by Seshadris ampleness criterion. There has been a lot of work on them over the years. We will give an overview of the research on Seshadri constants and discuss some important open questions.

Additivity of some multiplicative derivations on rings
Prof. Om Prakash, IIT Patna

Abstract: This talk presents a class of derivations that generalizes skew derivations and semi-derivations on a ring, and we call it skew semi-derivation. Furthermore, we discuss some of the conditions under which this type of multiplicative derivation becomes additive. Finally, to motive for further research, three different conditions for the additivity of a map on a triangular ring will be discussed with some open problems.

TBA
Prof. R.P. Shukla, University of Allahabad

Abstract: TBA

Primary Decomposition in S-Noetherian Modules
Prof. Shiv Datt Kumar, MNNIT Allahabad

Abstract: Development of the module theory is strongly influenced by the notion of Noetherian modules. One potential generalization of Noetherian modules is S-Noetherian modules. Let R be a commutative ring with identity and $S \subseteq R$ a multiplicative set, and Q be a submodule of an R-module M with $(Q :_R M) \cap S = \emptyset$, where $(Q :_R M) = \{r \in R \mid rM \subseteq Q\}$. Then Q is said to be an S-primary submodule if there exists $s \in S$ such that for all $a \in R$ and $m \in M$ if $am \in Q$, then either $sa \in \sqrt{(Q :_R M)}$ or $sm \in Q$.

An R-module M is called S-Noetherian if $sM \subseteq N$ for some finitely generated submodule N of M and some $s \in S$. We say that a submodule N of M with $(N :_R M) \cap S = \emptyset$ has an S-primary decomposition if it can be written as a finite intersection of S-primary submodules of M. The concept of S-primary submodules is a proper generalization of primary submodules. A natural way to extend primary decomposition from Noetherian modules to the larger class of S-Noetherian modules is to replace "primary submodules" by "S-primary submodules " in the decomposition process. In this talk, we introduce the concept of S-primary decomposition as a generalization of primary decomposition. First, we provide an example of an S-Noetherian module in which primary decomposition does not exist, which asserts that an S-Noetherian module need not be a Laskerian module in general. Then we establish the existence of S-primary decomposition in S-Noetherian modules as a generalization of historical Lasker-Noether decomposition theorem. We also prove S-version of first and second uniqueness theorems of primary decomposition for S-primary decomposition.

The primary decomposition of submodules gives an algebraic foundation for decomposing an algebraic variety into its irreducible components. Due to its significance, this theory evolved very quickly as one of the fundamental tools in commutative algebra and algebraic geometry. Several properties of S-primary submodules enjoys analogues of many properties of primary submodules.

From rings to matrix rings
Dr. Anuradha S Garge, University of Mumbai

Abstract: Many results are often studied by mathematicians over rings: commutative as well as non-commutative. There are two possible ways of generalizing these results: first, by understanding what properties of the ring are used. Second, by analyzing if the results may work for higher size matrices (results over rings being thought of as one-by-one matrices). In this talk, we shall see recent results in both these directions, which are joint work with students Naresh Afre, Murtuza Nullwala and Sandeep Kajabe.

Stability property of vector bundles
Dr. Suranto Basu, TCG Crest Kolkata

Abstract: The study of (algebraic) vector bundles is of fundamental importance in algebraic geometry. In this talk after briefly introducing basic properties of vector bundles we will discuss stability property of certain natural vector bundles associated to smooth , irreducible projective varieties defined over complex numbers. The talk will broadly be based on a joint work with Krishanu Dan. If time permits we will also discuss a recent result with Sarbeswar Pal.

Automorphisms of Product of Groups
Dr. Vipul Kakkar, Central University of Rajasthan

Abstract: In this talk, I will discuss automorphisms of different types of product of groups.

On Projective representations of groups
Dr. Sumana Hatui, NISER Bhubaneswar

Abstract: The theory of projective representations of groups, extensively studied by Schur, involves understanding homomorphisms from a group into the projective linear groups. By definition, every ordinary representation of a group is also projective, but the converse need not be true. Therefore, understanding a group's projective representations is a deeper problem and, many times, more difficult in nature. Two essential ingredients for studying the group's projective representations are describing its Schur multiplier and representation group. We will start with definitions and discuss several examples. We will also discuss some future research problems.

On a conjecture related to the Davenport constant
Dr. Renu Joshi, IISER Bhopal

Abstract: In this talk, we will discuss some problems in zero-sum theory, a branch of combinatorics where we look at G-sums for a finite group G. In 1963, Rogers defined a combinatorial invariant D(G) (Davenport constant), which is the minimum positive integer m such that every sequence of length m over a finite abelian group G has a non-trivial zero-sum subsequence. Olson and White examined this invariant in the case of a non-abelian group G, denoted by d(G) and Do(G) (according to the ordering of elements in a zero-sum sequence) for the first time in 1976, and found the upper bound for Do(G). Following that, D(G) was widely generalized; we will examine the significance of these generalizations and how they relate to one another. In 2004, Vesselin Dimitrov posed a conjecture that Do(G) and the Loewy length L(G) (nilpotency index of the augmentation ideal) are both equal for finite non-abelian p-groups. We provide the precise value of Do(G) and L(G) for a certain infinite family of non-abelian p-groups, which yields that the conjecture holds for this family of groups. For this infinite family of groups, the precise values of Do(G) also improve the existing upper bound for d(G).

Algebraic K-theory of Monoid Rings $R[M]$

Dr. Maria Mathew, IISER Pune

Abstract: My talk explores the evolution of classical K-theory in the context of monoid rings, stemming from the renowned Serres Problem (SP). Serres inquiry into whether projective modules over polynomial rings, with with coefficients coming from a field, are free or not, sparked the genesis of algebraic K-theory. I will trace the natural progression of K-theory in monoid rings, highlighting solved problems and identifying promising avenues for future exploration. I'll also discuss some convex geometrical connections that make these entities more intriguing and enrich our understanding of these structures.

Rationality of certain moduli spaces

Dr. Amit Kumar Singh, CMI Chennai

Abstract: The subject of algebraic geometry deals with understanding the geometry of zero sets of polynomials. The basic objects of interest in algebraic geometry are algebraic varieties, which locally look like the zero locus of a collection of polynomials. One of the very classical problems in this subject is the problem of classifying algebraic varieties up to isomorphism (fixing certain invariants). It turns out that it is very difficult to solve this problem completely. There is yet another classification problem unique to algebraic geometry: classifying algebraic varieties up to birational equivalence. In particular, it is to determine whether a given variety is birational to a projective space (such a variety is called a rational variety). Showing something is rational usually requires a lot of explicit geometric constructions that are classical in flavour. An affine space is the simplest example of a rational variety. More non-trivial examples of rational varieties are provided by certain 'moduli spaces'. A moduli space is a geometric space (usually a variety or scheme) whose points represent algebro-geometric objects of some fixed kind, or isomorphism classes of such objects. In this talk, we discuss the rationality questions of certain moduli spaces.

On completion of a gyrogroup

Dr. Sumit Kumar Upadhyay, IIIT Allahabad

Abstract: Gyrogroup is a non-associative algebraic structure, which is introduced by Ungar in 1988. The concept of gyrogroups first came into the picture in the study of Einsteins relativistic velocity addition law. In this talk, we will discuss an affirmative answer of the following question "For a given gyrogroup G, does there exists a pair $(M(G), \nu)$, where $M(G)$ is a group and ν is a gyrogroup homomorphism from G to $M(G)$ such that for any given group K and a gyrogroup homomorphism η from G to K, there exists a unique group homomorphism α from $M(G)$ to K such that $\alpha \circ \nu = \eta$?".

New Horizons in Gyrogroups: Semi-Cross Products and Extension Theory

Dr. Mani Shankar Pandey, IIITDM Kurnool

Abstract: In this talk, we present the concept of a semi-cross product between a group and a gyrogroup, offering a means to construct a broader range of gyrogroups. Additionally, we introduce a new class of gyrogroups that emerge from this construction. We further develop Schriers extension theory for gyrogroups, providing a novel framework for understanding the interactions between groups and gyrogroups. Consequently, we prove that a 2-fold extension $e \to H \to G \to K \to e$ of a group H by a gyrogroup K splits if and only if G is a semi-cross product of H and K.

Multiplicative Lie algebras and their Schur Multipliers

Mr. Amit Kumar, IIIT Allahabad

Abstract: It is evident that on a group (non-abelian) $(G, \cdot)$, there exist at least two distinct multiplicative Lie algebra structures. The first is the trivial structure defined as $x \star y = 1$ for all elements $x, y \in G$. The second structure is known as the improper multiplicative Lie algebra structure, defined as $x \star y = xyx^{-1}y^{-1}$. So, it is natural to ask for classification of all multiplicative Lie algebra structures on a given group. Pandey and Upadhyay did this classification for some finite multiplicative Lie algebras. Lal and Upadhyay developed the theory of extensions of multiplicative Lie algebras and introduced the Schur multiplier of multiplicative Lie algebras as second cohomology. In this talk, we will calculate the Schur multipliers of some finite multiplicative Lie algebras with respect to arbitrary multiplicative Lie algebra structures. Also, we see the limitations of our computations so that one can try to find some other way.

Some properties of tensor product of multiplicative Lie algebras

Mr. Deepak Pal, IIIT Allahabad

Abstract: In 2017, Donadze et al. introduced the concept of non-abelian tensor product of multiplicative Lie algebras. They proved that this notion recovers the existing notions of non-abelian tensor products of groups as well as of Lie algebras. In this talk, we discuss Lie nilpotent and Lie solvable properties of the non-abelian tensor product of multiplicative Lie algebras.